

Straight Lines

Question 1.

In a ΔABC , if A is the point (1, 2) and equations of the median through B and C are respectively $x + y = 5$ and $x = 4$, then B is

- (a) (1, 4)
- (b) (7, -2)
- (c) none of these
- (d) (4, 1)

Answer: (b) (7, -2)

The equation of median through B is $x + y = 5$

The point B lies on it.

Let the coordinates of B are $(x_1, 5 - x_1)$

Now CF is a median through C,

So coordinates of F i.e. mid-point of AB are

$$((x_1 + 1)/2, (5 - x_1 + 2)/2)$$

Now since this lies on $x = 4$

$$\Rightarrow (x_1 + 1)/2 = 4$$

$$\Rightarrow x_1 + 1 = 8$$

$$\Rightarrow x_1 = 7$$

Hence, the coordinates of B are (7, -2)

Question 2.

The equation of straight line passing through the point (1, 2) and perpendicular to the line $x + y + 1 = 0$

- (a) $y - x + 1 = 0$
- (b) $y - x - 1 = 0$
- (c) $y - x + 2 = 0$
- (d) $y - x - 2 = 0$

Answer: (b) $y - x - 1 = 0$

Given straight line is: $x + y + 1 = 0$

$$\Rightarrow y = -x - 1$$

Slope = -1

Now, required line is perpendicular to this line.

So, slope = $-1/-1 = 1$

Hence, the line is

$$y - 2 = 1 \times (x - 1)$$

$$\Rightarrow y - 2 = x - 1$$

$$\Rightarrow y - 2 - x + 1 = 0$$

$$\Rightarrow y - x - 1 = 0$$

Question 3.

The points $(-a, -b)$, $(0, 0)$, (a, b) and (a^2, ab) are

(a) vertices of a square

(b) vertices of a parallelogram

(c) collinear

(d) vertices of a rectangle

Answer: (c) collinear

Let the four points are $P(-a, -b)$, $O(0, 0)$, $Q(a, b)$ and $R(a^2, ab)$

Now,

$$m_1 = \text{slope of } OP = b/a$$

$$m_2 = \text{slope of } OQ = b/a$$

$$m_3 = \text{slope of } OR = b/a$$

Since $m_1 = m_2 = m_3$

So, the points O, P, Q, R are collinear.

Question 4.

The equation of the line through the points $(1, 5)$ and $(2, 3)$ is

(a) $2x - y - 7 = 0$

(b) $2x + y + 7 = 0$

(c) $2x + y - 7 = 0$

(d) $x + 2y - 7 = 0$

Answer: (c) $2x + y - 7 = 0$

Given, points are: $(1, 5)$ and $(2, 3)$

Now, equation of line is

$$y - y_1 = \{(y_2 - y_1)/(x_2 - x_1)\} \times (x - x_1)$$

$$\Rightarrow y - 5 = \{(3 - 5)/(2 - 1)\} \times (x - 1)$$

$$\Rightarrow y - 5 = (-2) \times (x - 1)$$

$$\Rightarrow y - 5 = -2x + 2$$

$$\Rightarrow 2x + y - 5 - 2 = 0$$

$$\Rightarrow 2x + y - 7 = 0$$

Question 5.

The slope of a line which passes through points (3, 2) and (-1, 5) is

- (a) $3/4$
- (b) $-3/4$
- (c) $4/3$
- (d) $-4/3$

Answer: (b) $-3/4$

Given, points are (3, 2) and (-1, 5)

Now, slope $m = (5 - 2)/(-1 - 3)$

$$\Rightarrow m = -3/4$$

So, the slope of the line is $-3/4$

Question 6.

The ratio of the 7th to the $(n - 1)$ th mean between 1 and 31, when n arithmetic means are inserted between them, is 5 : 9. The value of n is

- (a) 15
- (b) 12
- (c) 13
- (d) 14

Answer: (d) 14

Let the A.P. are 1, $A_1, A_2, A_3, \dots, A_m, 31$

$$a = 1, a_n = 31 \text{ and } n = m + 2$$

$$\text{Now, } a_n = a + (n - 1)d$$

$$\Rightarrow 31 = 1 + (m + 2 - 1)d$$

$$\Rightarrow 30 = (m + 1)d$$

$$\Rightarrow d = 30/(m + 1)$$

$$\text{Again, } A_7 = a + 7d = 1 + 7[30/(m + 1)] \dots\dots\dots 1$$

$$\text{and } A_{m-1} = a + (m - 1)d = 1 + (m - 1)[30/(m + 1)] \dots\dots\dots 2$$

From equation 1 and 2, we get

$$A_7/A_{m-1} = 5/9$$

$$\Rightarrow 1 + 7[30/(m + 1)] / 1 + (m - 1)[30/(m + 1)] = 5/9$$

$$\Rightarrow [m + 1 + 7(30)] / [m + 1 + 30m - 30] = 5/9$$

$$\Rightarrow [m + 211] / [31m - 29] = 5/9$$

$$\Rightarrow 9[m + 211] = 5[31m - 29]$$

$$\Rightarrow 9m + 1899 = 155m - 145$$

$$\Rightarrow 146m = 2044$$

$$\Rightarrow m = 2044/146$$

$$\Rightarrow m = 14$$

So, the value of m is 14

Question 7.

The ortho centre of the triangle formed by lines $xy = 0$ and $x + y = 1$ is :

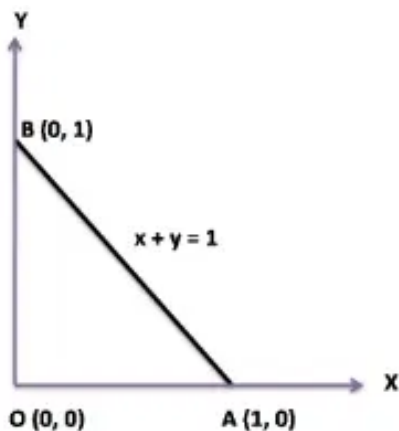
- (a) $(0, 0)$
- (b) none of these
- (c) $(1/2, 1/2)$
- (d) $(1/3, 1/3)$

Answer: (a) $(0, 0)$

Given lines are:

$$xy = 0 \text{ and } x + y = 1$$

$$\Rightarrow x = 0, y = 0 \text{ and } x + y = 1$$



In a triangle OAB, OA and OB are the altitudes which intersect at O.
So, the required orthocentre is $(0, 0)$

Question 8.

Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if

- (a) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
- (b) $a_1/a_2 \neq b_1/b_2 = c_1/c_2$
- (c) $a_1/a_2 \neq b_1/b_2 \neq c_1/c_2$
- (d) $a_1/a_2 = b_1/b_2 = c_1/c_2$

Answer: (a) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

Question 9.

If the line $x/a + y/b = 1$ passes through the points (2, -3) and (4, -5), then (a, b) is

- (a) $a = 1$ and $b = 1$
- (b) $a = 1$ and $b = -1$
- (c) $a = -1$ and $b = 1$
- (d) $a = -1$ and $b = -1$

Answer: (d) $a = -1$ and $b = -1$

Given equation of the line is $x/a + y/b = 1$

$$\Rightarrow bx + ay = ab$$

It is given that this line passes through (2, -3)

$$\Rightarrow b(2) + a(-3) = ab$$

$$\Rightarrow 2b - 3a = ab \text{ ——— (1)}$$

It also passes through (4, -5)

$$\Rightarrow 4b - 5a = ab \text{ ——— (2)}$$

On solving equation (1) and (2), we get

$$a = -1 \text{ and } b = -1$$

Question 10.

The angle between the lines $x - 2y = 5$ and $y - 2x = 5$ is

- (a) $\tan^{-1} (1/4)$
- (b) $\tan^{-1} (3/5)$
- (c) $\tan^{-1} (5/4)$
- (d) $\tan^{-1} (2/3)$

Answer: (c) $\tan^{-1} (5/4)$

Given, lines are:

$$x - 2y = 5 \text{ 1}$$

$$\text{and } y - 2x = 5 \text{ 2}$$

From equation 1,

$$x - 5 = 2y$$

$$\Rightarrow y = x/2 - 5/2$$

$$\text{Here, } m_1 = 1/2$$

From equation 2,

$$y = 2x + 5$$

$$\text{Here, } m_2 = 2$$

$$\text{Now, } \tan \theta = |(m_1 + m_2) / \{1 + m_1 \times m_2\}|$$

$$= |(1/2 + 2) / \{1 + (1/2) \times 2\}|$$

$$= |(5/2) / (1 + 1)|$$

$$= |(5/2) / 2|$$

$$= 5/4$$

$$\Rightarrow \theta = \tan^{-1} (5/4)$$

Question 11.

The points on the y-axis whose distance from the line $x/3 + y/4 = 1$ is 4 units is

- (a) $(0, 32/3)$ and $(0, 8/3)$
- (b) $(0, -32/3)$ and $(0, 8/3)$
- (c) $(0, -32/3)$ and $(0, -8/3)$
- (d) $(0, 32/3)$ and $(0, -8/3)$

Answer: (d) $(0, 32/3)$ and $(0, -8/3)$

Given equation of line is $(x/3) + (y/4) = 1$

$$\Rightarrow 4x + 3y = 12$$

$$\Rightarrow 4x + 3y - 12 = 0 \dots\dots\dots 1$$

Let $(0, b)$ is the point of the y-axis whose distance from given line is 4 unit.

When we compare equation 1 with general form of the equation $Ax + By + C = 0$, we get

$$A = 4, B = 3, C = -12$$

Now perpendicular distance of a line $Ax + By + C = 0$ from a point (x_1, y_1) is

$$d = |Ax_1 + By_1 + C|/\sqrt{(A^2 + B^2)}$$

So perpendicular distance of a line $4x + 3y - 12 = 0$ from a point $(0, b)$ is

$$4 = |4 \times 0 + 3 \times b - 12|/\sqrt{(4^2 + 3^2)}$$

$$\Rightarrow 4 = |3b - 12|/\sqrt{(16 + 9)}$$

$$\Rightarrow 4 = |3b - 12|/\sqrt{25}$$

$$\Rightarrow 4 = |3b - 12|/5$$

$$\Rightarrow 4 \times 5 = |3b - 12|$$

$$\Rightarrow |3b - 12| = 20$$

Now

$$3b - 12 = 20 \text{ and } 3b - 12 = -20$$

$$\Rightarrow 3b = 20 + 12 \text{ and } 3b = -20 + 12$$

$$\Rightarrow 3b = 32 \text{ and } 3b = -8$$

$$\Rightarrow b = 32/3 \text{ and } b = -8/3$$

So the points are $(0, 32/3)$ and $(0, -8/3)$

Question 12.

Equation of the line passing through $(0, 0)$ and slope m is

- (a) $y = mx + c$
- (b) $x = my + c$
- (c) $y = mx$
- (d) $x = my$

Answer: (c) $y = mx$

Equation of the line passing through (x_1, y_1) and slope m is

$$(y - y_1) = m(x - x_1)$$

Now, required line is

$$(y - 0) = m(x - 0)$$

$$\Rightarrow y = mx$$

Question 13.

The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is

(a) $3/10$

(b) $2/3$

(c) $3/2$

(d) $7/10$

Answer: (a) $3/10$

Given equations are:

$$3x + 4y = 9$$

$$\Rightarrow 3x + 4y - 9 = 0 \text{ and}$$

$$6x + 8y = 15$$

$$\Rightarrow 6x + 8y - 15 = 0$$

$$\Rightarrow 3x + 4y - 15/2 = 0$$

Now, compare these lines with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get

$$a_1 = 3, b_1 = 4, c_1 = -9 \text{ and}$$

$$a_2 = 3, b_2 = 4, c_2 = -15/2$$

Now, distance between two parallel line $= |c_1 - c_2| / \sqrt{a_1^2 + b_1^2}$

$$= |-9 + 15/2| / \sqrt{3^2 + 4^2}$$

$$= |(-18 + 15)/2| / \sqrt{25}$$

$$= |(-3/2)| / 5$$

$$= (3/2) / 5$$

$$= 3/10$$

Question 14.

What can be said regarding if a line if its slope is negative

(a) θ is an acute angle

(b) θ is an obtuse angle

(c) Either the line is x-axis or it is parallel to the x-axis.

(d) None of these

Answer: (b) θ is an obtuse angle

Let θ be the angle of inclination of the given line with the positive direction of x-axis in the

anticlockwise sense.

Then its slope is given by $m = \tan \theta$

Given, slope is positive

$$\Rightarrow \tan \theta < 0$$

$\Rightarrow \theta$ lies between 0 and 180 degree

$\Rightarrow \theta$ is an obtuse angle

Question 15.

Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if

(a) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

(b) $a_1/a_2 \neq b_1/b_2 = c_1/c_2$

(c) $a_1/a_2 \neq b_1/b_2 \neq c_1/c_2$

(d) $a_1/a_2 = b_1/b_2 = c_1/c_2$

Answer: (a) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

Question 16.

The slope of a line making inclination of 30° with the positive direction of x-axis is

(a) $1/2$

(b) $\sqrt{3}$

(c) $\sqrt{3}/2$

(d) $1/\sqrt{3}$

Answer: (d) $1/\sqrt{3}$

Here inclination of the line is 30°

So, slope of the line $m = \tan 30^\circ = 1/\sqrt{3}$

Question 17.

The perpendicular distance of a line $4x + 3y + 5 = 0$ from the point $(-1, 2)$ is

(a) 5

(b) 4

(c) 2

(d) 1

Answer: (c) 2

The perpendicular distance of a line $4x + 3y + 5 = 0$ from the point $(-1, 2)$

$$d = |4 \times (-1) + 3 \times 2 + 5|/\sqrt{(4^2 + 3^2)}$$

$$\Rightarrow d = |-4 + 9 + 5|/\sqrt{(16 + 9)}$$

$$\Rightarrow d = 10/\sqrt{(25)}$$

$$\Rightarrow d = 10/5$$

$$\Rightarrow d = 2$$

Question 18.

The inclination of the line $5x - 5y + 8 = 0$ is

(a) 30°

(b) 45°

(c) 60°

(d) 90°

Answer: (b) 45°

Given line is: $5x - 5y + 8 = 0$

$$\Rightarrow 5y = 5x + 8$$

$$\Rightarrow y = (5/5)x + 8/5$$

$$\Rightarrow y = x + 8/5$$

Now $\tan \theta = 1$

$$\Rightarrow \tan \theta = \tan 45^\circ$$

$$\Rightarrow \theta = 45^\circ$$

So, the inclination of the line is 45°

Question 19.

The points $(-a, -b)$, $(0, 0)$, (a, b) and (a^2, ab) are

(a) vertices of a square

(b) vertices of a parallelogram

(c) collinear

(d) vertices of a rectangle

Answer: (c) collinear

Let the four points are $P(-a, -b)$, $O(0, 0)$, $Q(a, b)$ and $R(a^2, ab)$

Now,

$$m_1 = \text{slope of } OP = b/a$$

$$m_2 = \text{slope of } OQ = b/a$$

$$m_3 = \text{slope of } OR = b/a$$

$$\text{Since } m_1 = m_2 = m_3$$

So, the points O, P, Q, R are collinear.

Question 20.

Given the three straight lines with equations $5x + 4y = 0$, $x + 2y - 10 = 0$ and $2x + y + 5 = 0$, then these lines are

- (a) none of these
- (b) the sides of a right angled triangle
- (c) concurrent
- (d) the sides of an equilateral triangle

Answer: (c) concurrent

Since the determinant of these lines is equal to zero

i.e.

$$\begin{vmatrix} 5 & 4 & 0 \\ 1 & 2 & -10 \\ 2 & 1 & -5 \end{vmatrix} = 0$$

So, these three lines are concurrent.
